

Forecasting and Optimization Approaches Utilized for Simulating a Hybrid District Heating Network

Lukas Gnam, Christian Pfeiffer, Markus Schindler, Markus Puchegger

Forschung Burgenland GmbH
Campus 1, 7000 Eisenstadt, Austria

Tel.: +43-5/7705-5475

lukas.gnam@forschung-burgenland.at

Abstract

The historically grown centralized energy system is undergoing massive changes due to the transformation from centralized energy production with large assets (e.g. fossil-thermal power plants) towards a sustainable, clean and decentralized energy system. This transformation is based on the inclusion of renewable energy sources (RESs) (e.g., wind and solar) into the classical systems. However, as the energy production stemming from RESs is extremely volatile and thus challenging to predict, new approaches have to be found in order to guarantee a successful integration of RESs into the existing infrastructure.

In the Austrian state of Burgenland approximately 1,000 MW of wind capacity is available. As already mentioned above, the high volatility of wind energy together with forecast uncertainties hinders the optimal integration of this RES into the existing energy system. Furthermore, the successful deployment of wind turbines was based on an attractive but timely limited subsidy scheme with a fixed feed-in tariff. As these subsidies now come to an end for more and more wind turbines and future support systems will rely on market premiums and tendering models, new approaches and business models have to be devised in order to sustain the rapid transformation of the classical energy systems.

In the research project HDH Demo in close cooperation with the city of Neusiedl am See, Burgenland, Austria, the aim is to integrate wind energy into the existing district heating grid of the city. This is realized by utilizing power-to-heat technologies, e.g., heat pumps. However, an economically feasible and successful integration is based on accurate forecasts for both, wind production and district heating demand as well as the actual energy prices.

Therefore, this work evaluates the applied data-driven forecasting methods. In particular, ensemble approaches that combine autoregressive models with artificial intelligent techniques are used to exploit the strengths of different methods (e.g. stability, flexibility). To compare the model performance, an overview on the accuracy and efficiency of the ensembles by using appropriate score metrics (e.g. RMSE, MAPE, R^2) is given. Furthermore, a mixed integer linear optimization model is presented for computing optimized schedules for the different components (e.g., heat pumps, energy storage units, biomass boiler) of the district heating grid. Together, these two approaches, forecasting and optimization, are used to investigate and evaluate different business models, which help to ensure the future market integration of wind production.

Keywords: forecasting, optimization, MILP, renewable energy sources

Introduction

The transformation of the centralized energy system relying mostly on fossil fuels towards decentralized and sustainable systems is based on the rapidly rising amount of production by renewable energy sources (RESs). One of the most prominent RES is wind energy, followed by photovoltaic (PV) systems. Contrary to fossil fuel based power plants, which are available at a moment's notice and can be adjusted accurately following the actual demand, RESs are characterized by totally different and highly volatile production characteristics. For example, wind turbines provide energy only in times of wind and PV systems rely on the incident solar radiation, which can vary highly depending on daytime, weather and cloud conditions. Subsequently, it is rather challenging to accurately predict the energy production stemming from RESs. Furthermore, it is possible that, depending on the weather, the production from RESs exceeds the predictions. In such situations the excess energy has to be sold at a very low or even negative price and/or is causing balancing energy costs directly affecting the economic viability of the specific renewable energy system, for instance, wind turbines.

In Austria the economic viability of wind turbines is heavily dependent on the subsidies (i.e., fixed feed-in tariffs) from the Austrian settlement agency for green energy (OeMAG). However, these subsidies are limited by law to 13 years resulting in economically challenging situations for wind park operators after the subsidy period [1]. Hence, it is economically and environmentally utterly important to develop new business models and thus keeping the wind turbines in operation. Otherwise, the ambitious goals of transforming the energy system is jeopardized due to economically unsuitable legal conditions. A long-term economic perspective for wind turbines affects not only the continuing operation of already existing systems but incentivizes also the construction of new modern wind parks, even for community-based systems.

The presented work focuses on the development of the aforementioned novel business models and their underlying technical and economic requirements, such as forecasting approaches and computational optimization models. These requirements are essential to evaluate prospective business models before testing them in a real-world test bed, i.e., the Austrian city Neusiedl am See. Neusiedl am See is a city in the eastern federal state of Burgenland, located directly at the northern shore of the lake Neusiedl. The city is a perfect test bed as there are numerous wind turbines located in its direct vicinity, which are already at the end of their subsidy period and are already marketed on the liberated electricity market.

Additionally, there is a district heating system in the city providing a test scenario for evaluating sector coupling options for the electric, the heating, and the gas system. The sector coupling allows for an integration of the RES wind into the district heating system of Neusiedl am See, for example, via heat pumps creating already one novel business model.

Hybrid Energy System of Neusiedl am See

The hybrid energy system of the city of Neusiedl am See is comprised of the electric system, consisting of the local wind parks, which are directly connected to an electric storage system (ESS) and four different heat pumps (HPs). The ESS is used as a backup solution for safely shutting down the HPs in cases where abrupt changes in wind power occur and thus insufficient energy for powering the HPs is available and for bridging short term undersupply. Two of the aforementioned HPs are air-to-water HPs (denoted as H_1 and H_2) delivering heat energy to a first thermal storage unit operating at low temperature (LT) (i.e., 30-40 °C and with a volume of around 14.000 liters). The remaining two HPs are water-to-water devices (H_3 and H_4) transferring energy from the first storage unit to a second thermal storage unit operating at 60-85 °C and containing about 18.000 liters. This second storage is directly

linked to a thermal buffer unit, which is providing the heating energy for the district heating grid. If the HPs alone cannot meet the district heating demand, several options exist to provide backup solutions: 1) A biomass boiler (BB) connected to 2) a flue gas condenser (FC), and 3) a gas burner (GB). The aforementioned components of the hybrid energy system operate in two different modes denoted as *summer* and *winter* mode. In summer mode, the air-to-water HPs directly provide energy to the low temperature storage unit, whereas in winter mode they transfer the energy directly to the thermal buffer unit. Furthermore, the BB and GB directly provide heating energy in winter mode for thermal buffer unit and the FC transfers heat to the low temperature storage unit. Additionally, the optimization model utilized in this study is able to include an electrolyzer into the hybrid energy system, extending the sector coupling options to power-to-gas applications. Figure 1 gives an overview of the different components of the hybrid energy system.

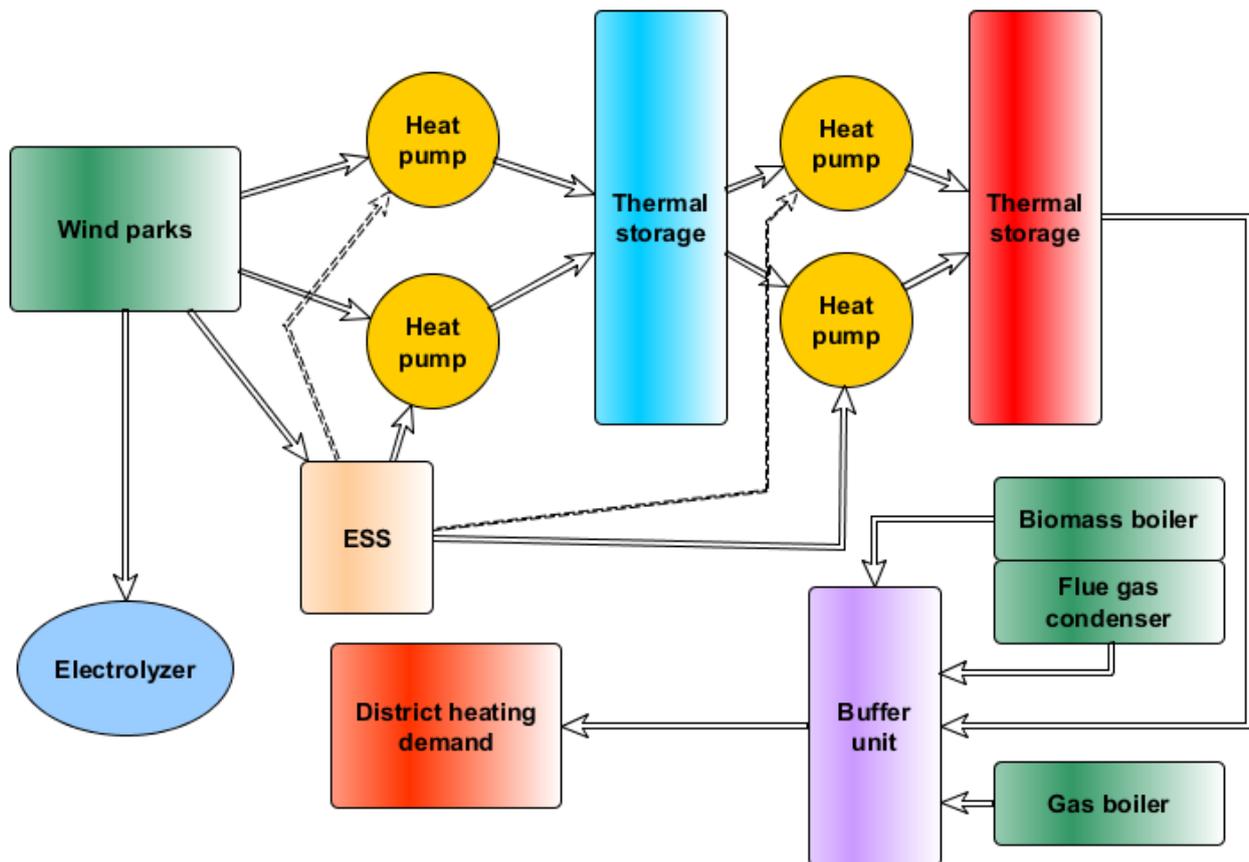


Figure 1 – Overview of the hybrid energy system of Neusiedl am See

Forecasting Approaches

To ensure an economically feasible and successful integration of RESs, accurate forecasts for district heating demand is needed. Research shows that the weather as well as the social behavior has most influence on the heat load in district heating grids [2]. The outdoor temperature as well as the humidity correlates the most with the heat demand in a district heating grid [3]. Hence, these parameters are chosen as inputs for the investigated models. The social behavior is taken into account implicitly by using the day of the week and the current month as an additional input [4]. To capture the dynamics of the buildings which are connected to the heating grid also past values of outdoor temperature and humidity are used. The similar but different model structures, i.e., AI and NARX models, are summarized in Figure 2.

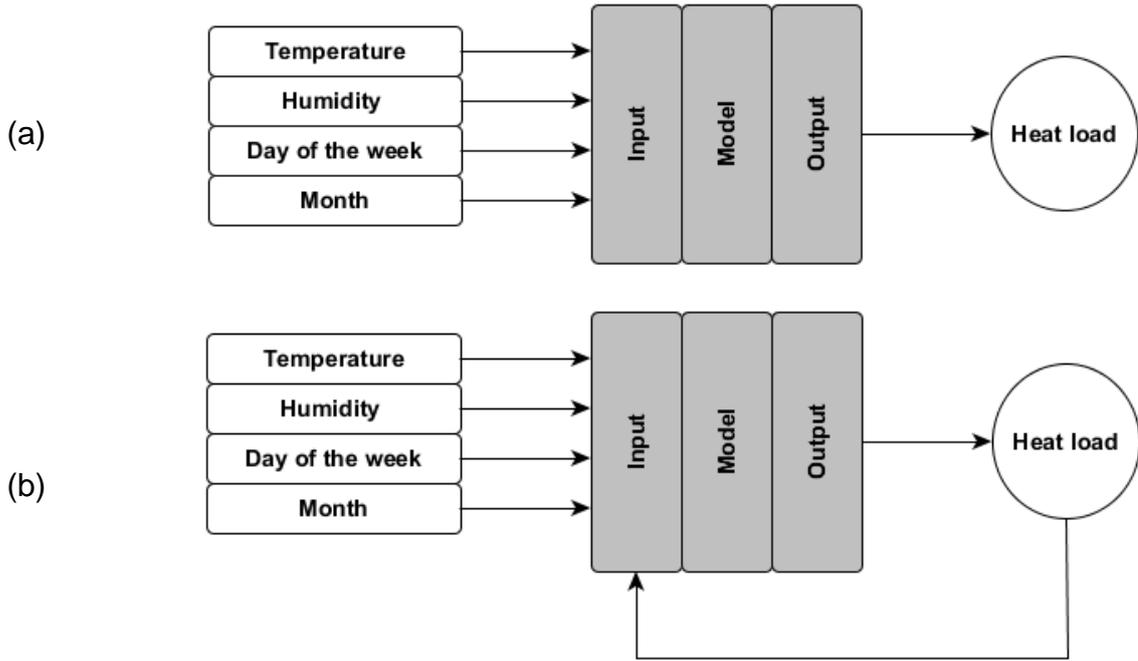


Figure 2 – Model structures of (a) AI and (b) NARX models

These models are used to generate realistic heat load forecasts for a specific time, based on weather data for the project region. In particular, three stationary artificial intelligence (AI) algorithms [5,6] i.e. Random Forest (RF) [7,8], k-Nearest Neighbor (k-NN) [9,10] and Artificial Neural Networks (ANN) [11,12], and a Nonlinear Autoregressive Exogenous (NARX) model [13] are utilized for creating the load models. Details on the models and their evaluation can be found in [14]. In this work, static ensemble approaches that combine these models to exploit their strengths due to flexibility of AI algorithms and stability of autoregressive models, i.e. mean ensemble (MENS), weighted mean ensemble (WENS), and seasonal weighted mean ensemble (SWENS) are used.

Static approaches assign a weight to each model in the ensemble, which is constant for all observations. MENS is the most common static approach, as it is the simple arithmetic mean of the predictions of the available models [15]. Furthermore, ensemble approaches, i.e., WENS and SWENS, which are weighted means and seasonal weighted means of the available models are applied. WENS uses the root mean squared error (see equation (1)) of the available models as weighting factor. SWENS additionally distinguishes between different seasonal errors.

The heat load was measured for a whole year in intervals of 15 minutes. 75% of the data are used as training set, whereas the remaining 25% of test data was split into four seasonal intervals. To compare the ensemble method performance, the metrics root mean squared error (*RMSE*), mean absolute error (*MAE*), and mean absolute percentage error (*MAPE*), as well as the coefficient of determination (R^2) are calculated, given by

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (\hat{y}_i - y_i)^2}, \quad (1)$$

$$MAE = \frac{1}{n} \sum_{i=1}^n |\hat{y}_i - y_i|, \quad (2)$$

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{\hat{y}_i - y_i}{y_i} \right|, \quad (3)$$

$$R^2 = 1 - \frac{\sum_{i=1}^n (\hat{y}_i - y_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}, \quad (4)$$

where \hat{y}_i is the predicted load, y_i is the observed load, \bar{y} is the mean of the observed loads, and n is the number of samples modeled.

Whilst the maximum heat demand in the district heating system is 785.70 kW, the average heat load of the network measured is 159.76 kW. As expected, the average heat load during summer is lower (68.64 kW) than in winter (409.51 kW).

Table 1 provides a comparison between the introduced ensemble models in terms of four statistical metrics that represent each model performance. In addition, the scores of k-NN as best performing individual model is listed as a baseline to compare the performance of the ensembles.

Table 1 – Evaluation of different approaches

Metric	Method	Test season				Overall
		Winter	Spring	Summer	Fall	
RMSE [kW]	k-NN	31.12	20.97	5.10	29.83	21.44
	MENS	29.73	18.06	4.34	28.49	19.60
	WENS	29.52	17.87	4.19	28.38	19.33
	SWENS	29.62	17.90	4.08	28.40	19.34
MAE [kW]	k-NN	17.72	10.36	3.14	17.29	11.05
	MENS	16.39	8.96	2.94	15.91	9.82
	WENS	16.03	8.75	2.82	15.76	9.54
	SWENS	16.19	8.78	2.67	15.78	9.51
MAPE [%]	k-NN	4.98	9.23	5.79	5.81	8.62
	MENS	4.17	8.40	4.51	4.65	6.91
	WENS	4.07	8.16	4.31	4.45	6.68
	SWENS	4.12	8.19	4.09	4.38	6.62
R ²	k-NN	.96	.95	.90	.94	.96
	MENS	.98	.97	.92	.97	.98
	WENS	.98	.98	.92	.97	.98
	SWENS	.98	.98	.93	.97	.98

Table 1 reveals that all ensemble approaches outperform the single k-NN model in terms of each measure. In winter and fall, comparatively high values of MAE are determined while values of MAPE indicate better forecast properties for winter. Due to a reduced heat demand, RMSE and MAE measures in spring and summer are comparatively small. In terms of MAPE, spring measures compared with winter and fall are even worse, indicating a poorer adequacy for prediction. Spring seems to be the most difficult season for predictions regarding heat demand.

Figure 3 indicates the performance of MENS and SWENS models in a one-week-time span in winter and spring. The graph shows that MENS (dotted orange line) yields slightly less accuracy than SWENS (dashed grey line) in both seasons. In winter MENS is not able to reach some of the peaks and tends to underestimate the average heat load. In comparison to the other models, the most precise model regarding the error measures appears to be SWENS as it catches the observed values in winter quite well. However, problems can be detected in cases of daily peaks in winter. On the one hand, viewing the accuracy on Saturday in the test week, heat demand is rather underestimated. On the other hand SWENS ignores lower heat load peaks on Wednesday and Thursday. The test week in

spring shows that MENS rather underestimates the daily peaks. In contrast, SWENS shows more accuracy in spring. However, some predictions still remain different from the observed values, confirming the reported error rates.

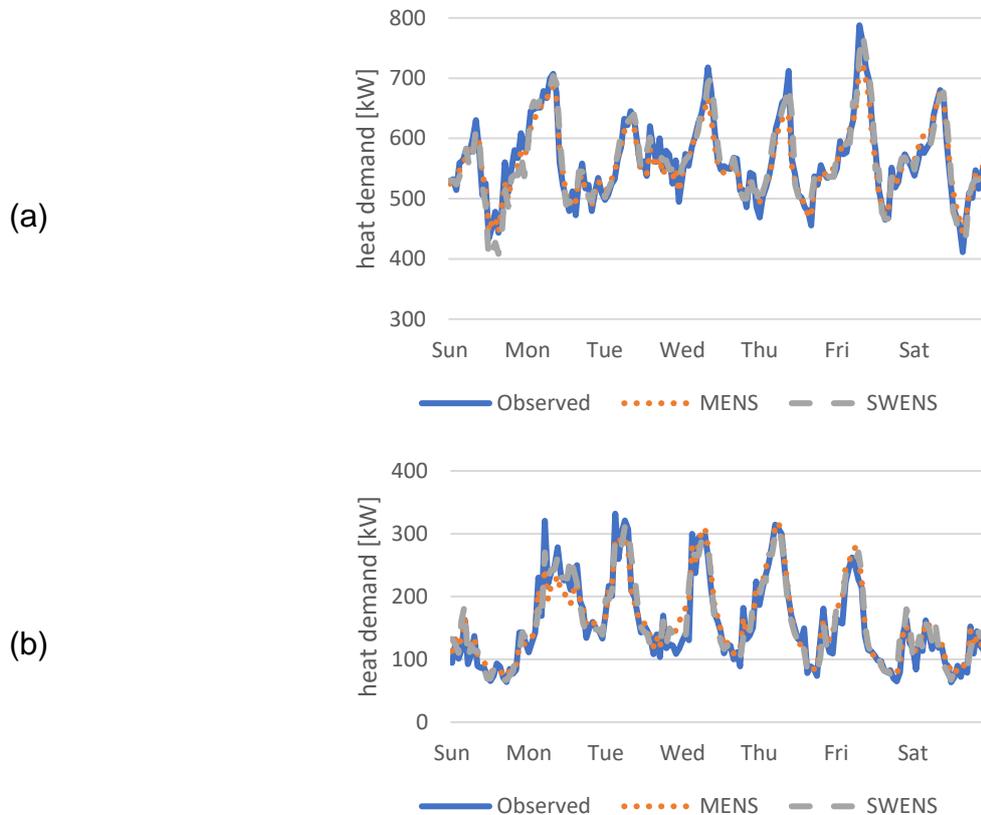


Figure 3 – Exemplary heat load values for a week in (a) winter and (b) spring.

Optimization Model

In order to model and simulate energy systems, various different approaches and methods exist. These range from using simple spreadsheet programs for small scale problems up to specifically designed modelling languages (e.g., GAMS [16], AMPL [17]) utilized to describe larger or more detailed systems. At the very core of each of these methods is the mathematical description of the system components. As the computational complexity is often directly affected by the number of components, it is of utmost importance to either keep the total number of modelled technical components as small as possible or to find simple but accurate mathematical descriptions of their behavior. The first approach is, however, not always possible as, for example, some systems of interest consist of numerous different components where their interaction is of major interest. Hence, different approaches for modelling the components are pursued in most of the scientific investigations. One prominent representative of these mathematical optimization or mathematical programming approaches is the so-called mixed integer linear programming method (MILP) [18]. Generally, a MILP optimization problem consists of an objective function and various constraints. The former is either minimized or maximized, whereas the latter define the value ranges for the variables. In a MILP formulation both, the objective function and the constraints, are linear with the particularity that some variables can only take integer values. For example, a variable denoting the status of a power plant can either be 0 or 1 for on and

off, respectively. However, despite their often relatively straightforward formulation, MILP optimization problems are NP-hard [18] and thus potentially require enormous computational resources for optimally solving the mathematical optimization problem, depending on the actual problem formulation.

In order to evaluate the hybrid energy system in the city of Neusiedl am See, a MILP optimization model [19] has been developed to include various different sector coupling approaches, e.g., HPs and electrolyzers. At the core of the model in the research project is the objective function [19]

$$\sum_{t=0}^T C(t) = \sum_{t=0}^T \left(\frac{p_f}{\eta_{BB}} \cdot P_{th, BB}(t) + p_{v, BB} \cdot \sigma_{BMP}(t) + c_{start, BB} \delta_{BMP}(t) + p_g \cdot P_{th, GB}(t) - p_{el} \varepsilon(t) - p_{h_2} \dot{m}_{pem}(t) + (p_{el} + p_{el, spread}) \cdot \left(P_{H1, el, grid}(t) + P_{H2, el, grid}(t) + P_{H3, el, grid}(t) + P_{H4, el, grid}(t) \right) \right) \quad (1).$$

Here, the first three terms describe the cost of the BB (biomass costs, variable costs and ramp up costs). The fourth term models the costs of the GB, and the following two terms reduce the overall system costs (i.e., selling excess energy and hydrogen). The last two terms model the electricity costs for the four different HPs.

As already mentioned above, the linear formulation of the objective function and constraints in a MILP optimization model is critical. Nevertheless, in reality there are numerous different examples of non-linear relationships. For example, the coefficient of performance (COP) value of a HP is denoted as

$$Q_{th} = COP(T) \cdot P_{el} \quad (2),$$

where $Q_{th}(t)$ is the thermal energy, $COP(T)$ the temperature dependent COP, and P_{el} the consumed electrical energy. Equation (2) is obviously a non-linear relationship and, thus, cannot be used as constraint in a MILP problem. Fortunately, utilizing a piecewise linear approximation, for example, as implemented in the *pyomo* framework [20], circumvents this particular issue. Although linearization is an approximation that comes with a certain error, choosing a suitable number of pieces for the piecewise approach proved to be a feasible option in preventing non-linearities in the MILP formulation of the hybrid energy system. For example, in Figure 4 two different linearization strategies are depicted for the summer and winter operation mode of one of the HPs in the hybrid energy system in Neusiedl am See. As the figure depicts, minor deviations between a simple linear approximation and the piecewise linear approach exist. Hence, for the given HP data the choice of the linearization method has negligible effect on the optimization outcome.

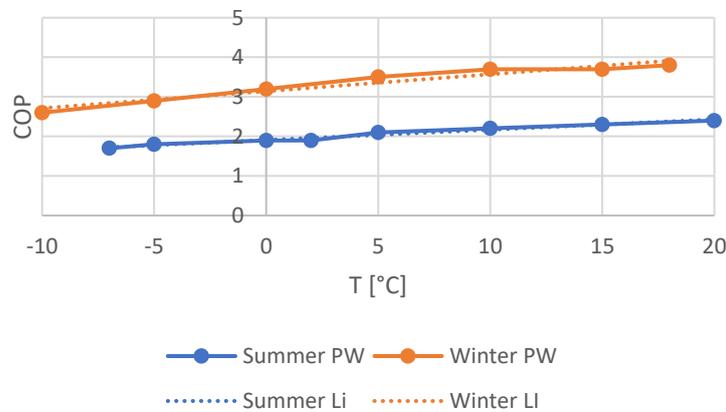


Figure 4 – Piecewise linearization (PW) and approximation using a linear function (LI) for an exemplary heat pump.

Another challenge occurring often while modelling energy systems is the description of minimal up- and down-times, for example, as it is the case for a BB. To formulate these constraints with linear equations the so-called Big-M method is utilized [18]. This formulation yields several inequalities given in equations (2)-(7).

$$switch_{BB}(t) = on_off_{BB}(t) \mid t = 0, \quad (2)$$

$$switch_{BB}(t) = on_off_{BB}(t) - on_off_{BB}(t - 1), \quad (3)$$

$$0 \leq -switch_{BB}(t) + M \cdot switch_{onBB}(t), \quad (4)$$

$$M \geq -switch_{BB}(t) + M \cdot switch_{onBB}(t) \quad (5)$$

$$0 \leq switch_{BB}(t) + M \cdot switch_{offBB}(t), \quad (6)$$

$$M \geq switch_{BB}(t) + M \cdot switch_{offBB}(t), \quad (7)$$

$switch_{BP}$ denotes the status of the switching operation at timestep t (i.e., -1 for switching off, 0 for no action and 1 for switching on) and the binary variable on_off_{BP} gives the status of the BB at a specific timestep. Furthermore, $switch_{onBB}$ and $switch_{offBB}$ are binary variables stating if a switching-on or -off operation is conducted at timestep t . Eventually, the minimum up- and down-times can now be formulated as constraints using sums over the binary variables $switch_{onBB}$ and $switch_{offBB}$.

Evaluation

In order to show the applicability of the forecasting and optimization approaches to the real-world test bed of Neusiedl am See, several different scenarios and business models for the hybrid district heating system of Neusiedl am See were set up. The first scenario utilizes only the HPs as consumers of the electric power provided by the regional wind parks. In the second scenario a 1.75 MW ESS is applied, whereas in the third scenario the ESS is replaced by a 17 MW electrolyzer as consumer. Finally, a fourth scenario with both, ESS and electrolyzer, is analyzed.

As first evaluation metric of the different scenarios the amount of excess energy is analyzed. This is the surplus energy from the regional wind parks around Neusiedl am See, which cannot be consumed in the hybrid district heating system. Figure 5 depicts the total excess energy over the course of a year. As the data indicates, the energy system extended with a 1.75 MW ESS experiences similar excess energy amounts as without an ESS. This is due to the fact that the capacity of the ESS is small compared to the amount of exceeding wind energy, particularly in times of high wind production. Additionally, for a real-world application the limited amount of charging cycles has to be taken into account. This particular property of ESSs is not reflected in the mathematical optimization model. Regarding the electrolyzer, the computational study shows that operation such a power-to-gas device reduces the amount of excess energy by a factor of about two. This seems quite promising, as the produced green hydrogen (i.e., production consuming electrical power originating only from renewable sources) can be used, for example, for regional public transport systems or sold to commercial customers. This alleviates the negative effects of dealing with unpredicted or enormously high excess energy on the balancing market. However, this simple analysis lacks a proper evaluation of the total expenditures for a power-to-gas technology would have to include operational expenditures (OPEX) as well as capital expenditures (CAPEX). As with today, these costs are one of the major obstacles of including hydrogen electrolyzers in existing energy systems. Fortunately, with further research effort and improvements the OPEX are going to decrease in the future. However, the CAPEX can still remain an issue, as high initial investments have to be amortized in a reasonable amount of time, otherwise electrolyzers will suffer from little to no economic attractiveness.

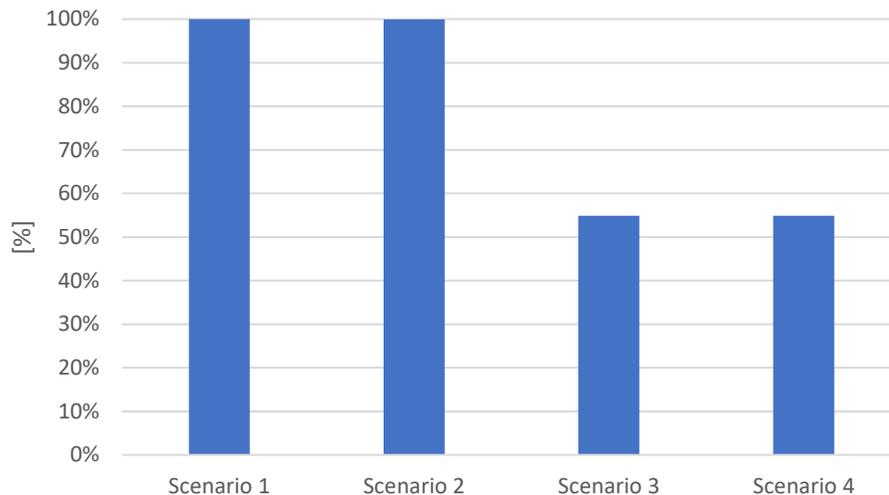


Figure 5 – Excess energy in the different scenarios.

Conclusions

The presented work shows how modern forecasting methods in their various forms together with mathematical optimization methods and techniques form a solid bases for a data-driven evaluation of future-fit energy systems, i.e., the hybrid district heating system of the Austrian city Neusiedl am See. A particular focus was lying on the integration of wind power via power-to-heat technologies, i.e., heat pumps, into the district heating system. However, a major share of wind energy has still to be sold on the regular energy market causing low or negative prices in some cases. Hence, different future technological options are evaluated, including the integration of an electrolyzer and an ESS. A computational analyses shows that utilizing a 17 MW electrolyzer reduces the amount of excess energy by a factor of about two, whereas a 1.75 MW ESS has negligible influence on the excess energy. Nevertheless, ESS are highly important components as they are used for safely shutting down wind power driven HPs when abrupt changes in wind power availability occurs. Furthermore, the work concludes that at the moment investment costs of electrolyzers pose the main challenge for including these power-to-gas technologies in existing energy grids. Therefore, political entities have to design appropriate and attractive subsidies or market structure to guarantee the ongoing transition of our classical energy systems. A successful implementation of funding opportunities helps to overcome the challenges of high initial costs for novel technologies as electrolyzers and, in addition, stimulate further research efforts in these directions. Together with novel business models and marketing strategies for unsubsidized wind energy the goal of a sustainable and fossil fuel free energy system can be successfully pursued.

Acknowledgements

The project “Hybrid DH Demo” is funded by the Austrian Climate and Energy Fund as part of the program “Smart Cities Demo”.

References

- [1] OeMAG. URL: <https://www.oem-ag.at/de/home/>. Accessed on 10 September 2020.
- [2] L. Arvastson. “Stochastic modelling and operational optimization in district-heating systems”. PhD Thesis, Sweden, 2001.

- [3] H. Yang, S. Jin, S. Feng, B. Wang, F. Zhang, J. Che. „Heat Load Forecasting of District Heating System Based on Numerical Weather Prediction Model”. *Proceedings of the 2nd International Forum on Electrical Engineering and Automation (IFEEA)*. 2015, pp 1-5. DOI: 10.2991/ifeea-15.20161.
- [4] H. Gadd, S. Werner. “Daily heat load variations in Swedish district heating systems”. *Applied Energy* 106 (2013), pp 47-55. DOI: 10.1016/j.apenergy.2013.01.030.
- [5] J. Antonanzas, N. Osorio, R. Escobar, R. Urraca, F.J. Martinez-de-Pison, F. Antonanzas-Torres. “Review of photovoltaic power forecasting”. *Solar Energy* 136 (2016), pp. 78-111. DOI: 10.1016/j.solener.2016.06.069
- [6] N. Fumo. “A review on the basics of building energy estimation”. *Renewable and Sustainable Energy Reviews* 31 (2014), pp. 53-60. DOI: 10.1016/j.rser.2013.11.040.
- [7] L. Breiman. “Random Forests”. *Machine Learning* 45 (2001), pp. 5-32. DOI: 10.1023/A:1010933404324.
- [8] G. Dudek. “Short-Term Load Forecasting Using Random Forests”. *Proceedings of the 7th IEEE International Conference Intelligent Systems (IS'2014)*. 2014, pp 821-828. DOI: 10.1007/978-3-319-11310-4_71
- [9] S. Yakowitz. “Nearest-Neighbour Methods for Time Series Analysis”. *Journal of Time Series Analysis* 8.2 (1987), pp. 235-247. DOI: 10.1111/j.1467-9892.1987.tb00435.x.
- [10] F.H. Al-Qathani, S.F., Crone. “Multivariate k-Nearest Neighbour Regression for Time Series data – a novel Algorithm for Forecasting UK Electricity Demand”. *Proceedings of International Joint Conference on Neural Networks (IJCNN)*. 2013, pp. 228-235. DOI: 10.1109/IJCNN.2013.6706742.
- [11] C.M. Bishop. *Neural Networks for Pattern Recognition*. Oxford University Press, 1997. ISBN: 0198538642.
- [12] M. Hayati, Y. Shirvany. “Artificial Neural Network Approach for Short Term Load Forecasting for Illam Region”. *International Journal of Electrical, Computer, and Systems Engineering* 1.2 (2007), pp. 121-125. DOI: 10.5281/zenodo.1328642.
- [13] L.G.B. Ruiz, M.P. Cuéllar, M.D. Calvo-Flores, M.D. Jiménez. “An Application of Non-Linear Autoregressive Neural Networks to Predict Energy Consumption in Public Buildings”. *Energies* 9.9 (2016), p. 684. DOI: 10.3390/en9090684.
- [14] G. Steindl, C., Pfeiffer. “Comparison of Black Box Models for Load Profile Generation of District Heating Networks”. *Proceedings of 12th Conference on Sustainable Development of Energy, Water and Environment Systems (SDEWES)*. 2017.
- [15] R.T., Clemen. “Combining forecasts: A review and annotated bibliography”. *International journal of forecasting* 5.4 (1989), pp. 559–583. DOI: 10.1016/0169-2070(89)90012-5.
- [16] GAMS Development Corporation. “General Algebraic Modeling System (GAMS) Release 27.1.0”. Fairfax, VA, USA, 2019.
- [17] R. Fourer, D.M. Gay, B.W. Kernighan. *AMPL: A Modeling Language for Mathematical Programming*. 2nd ed. Cengage Learning, 2002. ISBN: 0534388094.
- [18] G. Sierksma, Y. Zwols. *Linear and Integer Optimization*. CRC Press, 2015. ISBN: 9781498743129.
- [19] M. Schindler, L. Gnam, T. Nacht. „A simulation study on the integration of wind in a district heating system“. *e-Nova International Conference on Sustainable Technologies (e-Nova)*. 2020. In press.
- [20] W.E. Hart, C. D. Laird, J.-P. Watson, D.L. Woodruff, G.A. Hackebeil, B.L. Nicholson, J. D. Sirola. *Pyomo – Optimization Modeling in Python*. Springer International Publishing, 2017. ISBN: 9783319588193.